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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MS2B Statistics 2

Mark Scheme

2007 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

$\surd$ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	C	candidate
PI	possibly implied	Sf	significant figure(s)
SCA	substantially correct approach	Dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

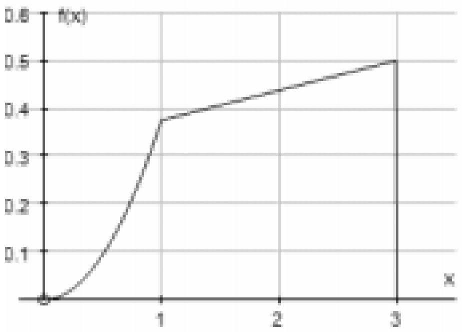
MS2B

Q	Solution	Marks	Total	Comments
1	$\bar{x} = 39.5 \quad s = 4.84 \quad (s^2 = 23.4)$	B1B1		$\sigma = 4.53 \quad (\sigma^2 = 20.5)$
	$t_{\text{crit}} = 2.365$	B1		
	95% CI for μ			
	$= \bar{x} \pm t_{\text{crit}} \times \dfrac{s}{\sqrt{n}}$			
	$= 39.5 \pm 2.365 \times \dfrac{4.84}{\sqrt{8}}$	M1		$39.5 \pm 2.365 \times \dfrac{4.53}{\sqrt{7}}$
	$= 39.5 \pm 4.05$			
	$= (35.5, 43.5)$	A1√	5	
	Total		**5**	
2(a)(i)	$\mathrm{P}(A = 4) = \dfrac{\mathrm{e}^{-3.5} \times (3.5)^4}{4!} = 0.189$	M1A1	2	
(ii)	$\mathrm{P}(B \le 6) = 0.762$	B1	1	
(iii)	$T = A + B \sim \mathrm{Po}(8.5)$			
	$\mathrm{P}(T \text{ fewer than } 10) = \mathrm{P}(T < 10)$	M1		Use of Po (8.5)
	$= \mathrm{P}(T \le 9)$	M1		$T \le 9$ attempted
	$= 0.653$	A1	3	CAO
(b)	$X \sim \mathrm{B}(5, 0.653)$	B1		$X \sim \mathrm{B}(5, \text{their } p)$
	$\mathrm{P}(X \ge 4) = \dbinom{5}{4}(0.653)^4(0.347)$			
	$+ (0.653)^5$	M1		
	$= 0.31547 + 0.11873$			
	$= 0.434$	A1√	3	On their p from (a)(iii)
(c)(i)	$\bar{x} = 9.2$	B1		
	$s^2 = 9.29$	B1	2	$\sigma^2 = 8.36$
(ii)	Mean and variance have similar values which suggests that Poisson distribution may be appropriate	B1√ B1√	2	
	Total		**13**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
3	$\bar{x} = 83.5$	B1		
	$s^2 = \frac{1}{99}(15321) = 154.76$			$(154 < s^2 \le 155)$
	$s = 12.44$	B1		$(12.4 \le s \le 12.45)$
	$H_0 : \mu = 85.9$ $H_1 : \mu \ne 85.9$	B1		
	Under H_0, $\bar{X} \sim N\left(85.9, \frac{(12.44^2)}{100}\right)$			
	$z_{crit} = \pm 1.96$	B1		$z = 1.96 +$ 2 tail test used
	$z = \frac{83.5 - 85.9}{12.44/10} = -1.929$	M1		$\frac{(\text{their } \bar{x}) - 85.9}{(\text{their } s)/10}$
		A1		AWFW –1.94 to 1.92
	accept H_0, reject the claim	A1√		On their z
	Insufficient evidence to suggest that the mean has changed from 85.9 at the 5% level of significance.	E1√	8	
	Total		**8**	
4(a)	$\sum p = 1$ $k = 1 - (0.01 + 0.05 + 0.14 + 0.30 + 0.12)$ $k = 0.38$	B1	1	
(b)(i)	$E(X) = \sum_{\text{all } x} x\, P(X = x) = 4.35$	B1	1	$\frac{87}{20}$
(ii)	$\text{Var}(X) = \sum_{\text{all } x} x^2\ P(X = x) - \mu^2$	M1		$E(X^2)$ attempted
	$= 20.09 - 18.9225$	M1		$\sum x^2 P(X = x) - \mu^2$
	$= 1.1675$	A1	3	$\frac{467}{400}$ (AWFW 1.16 – 1.17)
(c)(i)	$E(Y) = 5E(X) + 2$ $= 5 \times 4.35 + 2$ $= 23.75$	M1	1	Their (b)(i) $\times 5 + 2$
(ii)	$\text{Var}(Y) = 25\text{Var}(X)$ $= 29.1875$	M1		Their (b)(ii) $\times$ 25
	Standard deviation $= 5.40$	m1		$\sqrt{\ }$
		A1	3	$(5.40 - 5.41)$
	Total		**9**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\text{H}_0 : \mu = 30$			
	$\text{H}_1 : \mu > 30$	B1		
	$\bar{x} = 33.5$ and $s = 4.25$ $\left(s^2 = 18.06\right)$	B1B1		$\sigma = 4.03$ $\left(\sigma^2 = 16.25\right)$
	Under H_0 $\bar{X} \sim \text{N}\left(30, \dfrac{\left(4.25^2\right)}{10}\right)$			$\downarrow$
	$t = \dfrac{33.5 - 30}{4.25/\sqrt{10}} = 2.60$	M1A1		$\dfrac{33.5 - 30}{4.03/\sqrt{9}}$ $\left(2.6 - 2.61\right)$
	$t_{\text{crit}} = 2.821$	B1		
	do not reject H_0			
	Insufficient evidence at the 1% level of significance that Jasmine's teacher is underestimating the time that it takes to complete the homework assignments.	E1√	7	
(b)	Times are Normally distributed	B1	1	
	Total		**8**	
6(a)		B1 B1 B1	3	for curve for line for axes
(b)	$\text{P}\left(T \geq 1\right) = \dfrac{1}{2} \times \dfrac{7}{8} \times 2 = \dfrac{7}{8}$	M1A1	2	OE

MS2B (cont)

Q	Solution	Marks	Total	Comments
6(c)(i)	For $1 \le t \le 3$			
	$\int_1^t \frac{1}{16}(t+5)\,\mathrm{d}t = \left[\frac{1}{32}t^2 + \frac{5}{16}t\right]_1^t$	M1A1		
	$\mathrm{F}(1) = \frac{1}{8}$	B1		
	$\mathrm{F}(t) = \frac{1}{8} + \frac{1}{32}t^2 + \frac{5}{16}t - \frac{11}{32}$	M1		Use of: $\mathrm{F}(t) = \mathrm{F}(1) + \int_1^t \frac{1}{16}(t+5)\,\mathrm{d}t$
	$\mathrm{F}(t) = \frac{1}{32}(t^2 + 10t - 7)$	A1	5	**AG**
	Alternative:			
	$\int \frac{1}{16}(t+5)\,\mathrm{d}t$			
	$= \frac{1}{16}\left(\frac{1}{2}t^2 + 5t + c\right)$	(M1) (A1)		
	$\mathrm{F}(1) = \frac{1}{8}$	(B1)		
	$\Rightarrow\ c = -3.5$	(M1)		
	$\mathrm{F}(t) = \frac{1}{32}(t^2 + 10t - 7)$	(A1)		
(ii)	$\frac{1}{32}(m^2 + 10m - 7) = 0.5$	M1		
	$m^2 + 10m - 23 = 0$	A1		
	$m = \frac{-10 \pm \sqrt{192}}{2} = -5 \pm \sqrt{48}$ $= -5 \pm 4\sqrt{3}$	m1		(or any valid method)
	$(m > 0)$			
	$m = 4\sqrt{3} - 5 = 1.93$	A1	4	(1.9282)
	Total		**14**	

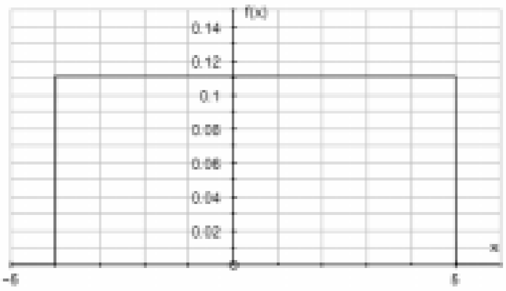
MS2B (cont)

Q	Solution	Marks	Total	Comments
7(a)	H_0 : No association between the performances at KS3 and GCE	B1		

O_i	E_i	$O_i - E_i$	X^2	Marks	Comments
60	63.55	–3.55	0.1983		
55	44.64	10.36	2.4043	M1	E_i
40	46.81	–6.81	0.9907		
				M1	O_i - E_i
55	51.25	3.75	0.2744		
32	36.00	–4.00	0.4444		
38	37.75	0.25	0.0017	M1	$(O_i - E_i)^2 / E_i$
47	46.33	0.67	0.0097	M1	$\sum$
31	32.54	–1.54	0.0733		
35	34.13	0.87	0.0222		
43	43.87	–0.87	0.0173		
26	30.82	–4.82	0.7527		
38	32.31	5.69	1.0005		

Q	Solution	Marks	Total	Comments
	$X^2 = 6.1897$	A1		AWFW 6.05 – 6.35
	$\nu = 3 \times 2 = 6 \quad \Rightarrow \quad \chi^2_{90\%} = 10.645$	B1B1√		on their ν
	Do not reject H_0 No evidence to suggest an association between KS3 results and GCE grades at 10% level of significance.	E1√	9	
(b)	More of the students achieving level 7 at KS3 gain grade A's at GCE than expected.	E1	1	
	Total		**10**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
8(a)	$f(x)=\begin{cases}\frac{1}{9} & -4\le x\le 5\\ 0 & \text{otherwise}\end{cases}$	M1 A1	2	
(b)		B1 B1	2	horizontal line from –4 to 5 for drawn at $\frac{1}{9}$
(c)	$P(X>2)=\frac{1}{9}\times 3$ $=\frac{1}{3}$	M1 A1	2	$F(5)-F(2)$ $=1-\frac{2}{3}$ $=\frac{1}{3}$
(d)	Mean $=\frac{1}{2}$ Variance $=\frac{1}{12}\times 81$ $=6.75$	B1 B1	2	
	Total		**8**	
	TOTAL		**75**	